Test -4 solutions.

M76
Burger.

#1.

$$\sum_{n=0}^{\infty}\left(\tfrac{1}{8}\right)^n - \sum_{n=0}^{\infty}\left(-\tfrac{1}{8}\right)^n$$

$|-\frac{1}{8}| < 1$

$|\frac{1}{8}| < 1$

$$\frac{1}{1-\frac{1}{8}} - \frac{1}{1+\frac{1}{8}}$$

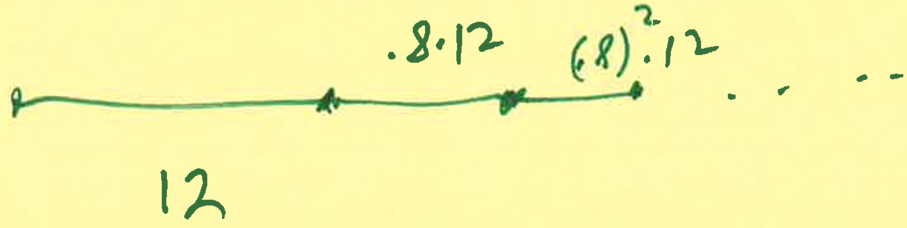
$$\frac{1}{\frac{7}{8}} - \frac{1}{\frac{9}{8}}$$

$$\frac{8}{7} - \frac{8}{9}$$

$$\frac{72-56}{63}$$

$$\frac{16}{63} \rightsquigarrow \underline{D}$$

#2.



$$(*) = \sum_{n=0}^{\infty} 12\left(\frac{8}{10}\right)^n$$

$\frac{8}{10} = \left|\frac{4}{5}\right| < 1$

convergent geometric series.

$$(*) = \frac{12}{1-4/5} = \frac{12}{\frac{1}{5}} = 12\cdot 5 = 60\text{ m} \rightsquigarrow C$$

#3. Write out some terms:

$$1 - \cancel{\frac{1}{\sqrt{8}}} + \cancel{\frac{1}{\sqrt{8}}} - \cancel{\frac{1}{\sqrt{27}}} + \cancel{\frac{1}{\sqrt{27}}} - \cdots$$

$$\rightarrow 1 \rightsquigarrow \text{D}$$

#4. Comparison Test.

$$0 < \frac{1}{4^{n-1}+1} < \underbrace{\frac{1}{4^{n-1}}}_{\frac{1}{4}\left(\frac{1}{4}\right)^n} < \left(\frac{1}{4}\right)^n$$

An even easier way to do this.

Thus $\sum_{n=1}^{\infty} \frac{1}{4^{n-1}+1}$ converges

by comparison to $\rightsquigarrow$ A convergent geometric series $\left|\frac{1}{4}\right| < 1$

(SQUEEZE Theorem)

A

#5. Integral Test.

$$\lim_{b\to\infty} \int_1^b \frac{3x}{x^2+3}\,dx$$

let $u = x^2+3$

$du = 2x\,dx$

$\frac{3}{2} du = 3x\,dx$

$$\frac{3}{2}\int \frac{du}{u} = \frac{3}{2}\ln(x^2+3)$$

$$\frac{3}{2} \cdot \lim_{b\to\infty} \ln(x^2+3) \Big|_1^b = \lim_{b\to\infty} \frac{3}{2}\ln(b^2+3) - \frac{3}{2}\ln 4$$

$\to \infty$

ln is increasing function.

$\infty \implies$

Diverges.

A.

#6. Ratio Test.

$$\lim_{n\to\infty}\left|\frac{\dfrac{(n+1)^4}{4^{n+1}}}{\dfrac{n^4}{4^n}}\right| =$$

$$\lim_{n\to\infty}\left|\frac{(n+1)^4\,4^n}{n^4\,4^{n+1}}\right| =$$

$$\lim_{n\to\infty}\left(\frac{n+1}{n}\right)^4\cdot\frac{1}{4} = \lim_{n\to\infty}\left(\frac{1+\overset{0}{\frac{1}{n}}}{1}\right)^4\cdot\frac{1}{4}$$

$$= \frac{1}{4} < 1$$

Converges.

B

7. $\sum \frac{(-1)^n}{2n^{1/3}+6}$.

Alt. Series Test:

$\lim_{n\to\infty} \frac{1}{2n^{1/3}+6} \rightsquigarrow 0$ converges.

$\therefore$ passes.

Atleast conditional at this point.

Now: $\sum_{n=1}^{\infty} \frac{1}{2n^{1/3}+6}$

(C)

Limit Comparison Test

w/ ~~divergent~~ p-series $\frac{1}{n^{1/3}}$

$$\lim_{n\to\infty} \frac{\frac{1}{2n^{1/3}+6}}{\frac{1}{n^{1/3}}} =$$

$$\lim_{n\to\infty} \frac{n^{1/3} \cdot \frac{1}{n^{1/3}}}{(2n^{1/3}+6)\,\frac{1}{n^{1/3}}} = \lim_{n\to\infty} \frac{1}{2+\cancel{\frac{6}{n^{1/3}}}_{\to 0}} = \frac{1}{2} > 0$$

So Diverges And <u>NOT</u> Absolutely convergent. $\therefore$ (C)

#8.

Alt. series test.

$$\lim_{n\to\infty} \frac{2^n}{(8n^4+2^n)} \cdot \frac{1/2^n}{1/2^n} = \lim_{n\to\infty} \frac{1}{\frac{8n^4}{2^n}+1}$$

$\frac{8n^4}{2^n} \to 0$ By speed of convergences.

$$\frac{(-2)^n}{8n^4+2^n} = \frac{(-1)^n 2^n}{8n^4+2^n}$$

$= 1 \neq 0$

Hence it is a Divergent Alt. series.

A

9.

Divergence Test.

(Recall speed of convergence inequalities)

$(\ln k)^p < k$

$$\lim_{k\to\infty} \frac{k}{(\ln k)^{30}} \rightsquigarrow \frac{1}{30(\ln k)^{29}\cdot\frac{1}{k}}$$

repeated L'Hopital's Rule!

$\frac{\infty}{\infty}$

$$\rightsquigarrow \frac{k}{30(\ln k)^{29}} \longrightarrow \lim_{k\to\infty} \frac{1}{30\cdot 29(\ln k)^{28}\cdot\frac{1}{k}}$$

$\frac{\infty}{\infty}$

After doing this many times!

(L'Hopitals Rule)

$$\rightsquigarrow \frac{k}{30\cdot 29\cdots 2\ln k} \xrightarrow{\text{1 more time.}} \frac{1}{30!\cdot\frac{1}{k}} =$$

$\frac{\infty}{\infty}$

$$\boxed{\lim_{k\to\infty} \frac{k}{30!} \to \infty}$$

Diverses

Series Diverses By Divergence test!

A

#10 Let $\frac{6}{n(n+1)(n+2)} = \frac{A}{n} + \frac{B}{n+1} + \frac{C}{n+2}$

then $6 = A(n+1)(n+2) + B(n+2)\cdot n + C\cdot(n+1)\cdot n$

Arnold's Method!

$n=0 \rightsquigarrow A(1)(2) = 6 \rightarrow \boxed{A=3}$

$n=-1 \rightsquigarrow B(-1+2)(-1) = -B = 6 \rightsquigarrow \boxed{B=-6}$

$n=-2 \rightsquigarrow C(-2+1)(-2) = 2C = 6 \rightsquigarrow \boxed{C=3}$

Need $\sum_{n=1}^{\infty} \frac{3}{n} - \frac{6}{n+1} + \frac{3}{n+2}$

$= \cancel{\frac{3}{1} - \frac{6}{2}} + \cancel{\frac{3}{3}} + \frac{3}{2} - \cancel{\frac{6}{3}} + \cancel{\frac{3}{4}} + \cancel{\frac{3}{3}} - \cancel{\frac{6}{4}} + \cancel{\frac{3}{5}}$

$+ \cancel{\frac{3}{4}} - \cancel{\frac{6}{5}} + \cancel{\frac{3}{6}} + \cancel{\frac{3}{5}} - \cancel{\frac{6}{6}} + \cancel{\frac{3}{7}} + \cancel{\frac{3}{6}} - \cancel{\frac{6}{7}} + \cancel{\frac{3}{8}} +$

$\cancel{\frac{3}{7}} - \cancel{\frac{6}{8}} + \cancel{\frac{3}{9}} + \cancel{\frac{3}{8}} - \cancel{\frac{6}{9}} + \cancel{\frac{3}{10}} + \cancel{\frac{3}{9}} \ldots - \cancel{\frac{6}{10}} + \cancel{\frac{3}{11}} \ldots$

they all cancel from here!

Converges to $\frac{3}{2}$ D